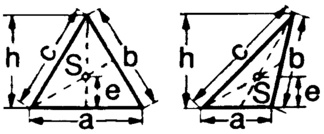
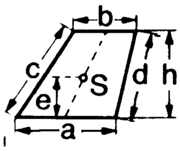
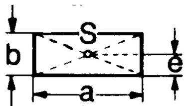
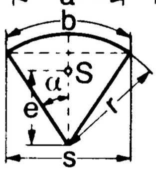
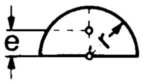
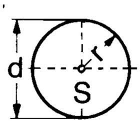
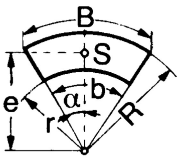
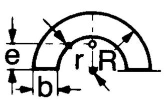
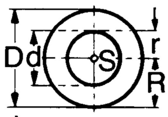
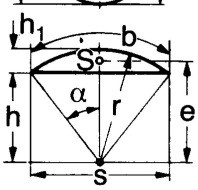
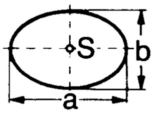
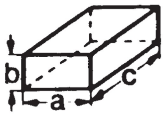
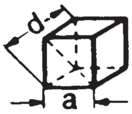
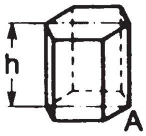
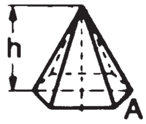
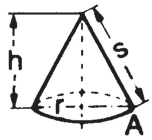
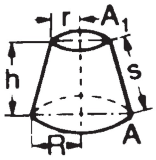
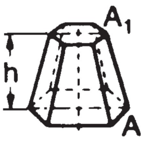
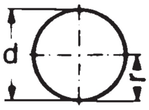
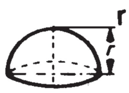
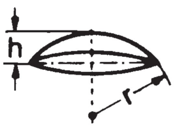
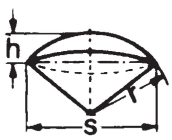
1.4.2 Areas and centres of gravity

Table 1-24

Shape of surface		A = area	U = perimeter S = centre of gravity (cg) e = distance of cg
Triangle		$A = \frac{1}{2}\,a\,h$	$U = a + b + c$ $e = \frac{1}{3}\,h$
Trapezium		$A = \frac{a + b}{2} \cdot h$	$U = a + b + c + d$ $e = \frac{h}{3} \cdot \frac{a + 2\,b}{a + b}$
Rectangle		$A = a\,b$	$U = 2\,(a + b)$
Circle segment		$A = \frac{b\,r}{2} = \frac{\alpha^0}{180}\,r\,\pi$ $b = r\,\pi\,\frac{\alpha^0}{180}$	$U = 2\,r + b$ $e = \frac{2}{3}\,r\,\frac{\sin\alpha}{\alpha^0} \cdot \frac{180}{\pi}$
Semicircle		$A = \frac{1}{2}\,\pi\,r^2$	$U = r\,(2 + \pi) = 5.14\,r$ $e = \frac{1}{3} \cdot \frac{r}{\pi} = 0.425\,r$
Circle		$A = r^2\,\pi = \pi\,\frac{d^2}{4}$	$U = 2\,\pi\,r = \pi\,d$
Annular segment		$A = \frac{\pi}{180}\,\alpha^0\,(R^2 - r^2)$	$U = 2\,(R - r) + B + b$ $e = \frac{2}{3} \cdot \frac{R^2 - r^2}{R^2 - r^2} \cdot \frac{\sin\alpha}{\alpha^0} \cdot \frac{180}{\pi}$
Semi-annulus		$A = \frac{\pi}{2}\,\alpha^0\,(R^2 - r^2)$	if $b < 0.2\,R$, then $e \approx 0.32\,(R + r)$
Annulus		$A = \pi\,(R^2 - r^2)$	$U = 2\,\pi\,(R + r)$
Circular segment		$A = \frac{\alpha^0}{180}\,r^2\,\pi - \frac{s\,h}{2}$ $s = 2\,\sqrt{r^2 - h^2}$	$U = 2\,\sqrt{r^2 - h^2} + \frac{\pi\,r\,\alpha^0}{90}$ $e = \frac{s^2}{12 \cdot A}$
Ellipse		$A = \frac{a\,b}{4}\,\pi$	$U = \frac{\pi}{2}\left[\,1.5\,(a + b) - \sqrt{ab}\,\right]$

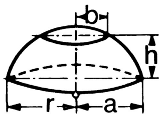
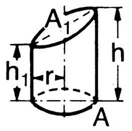
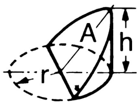
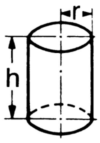
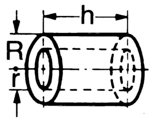
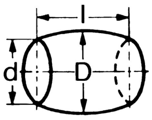
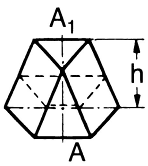
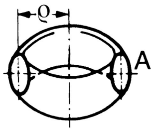
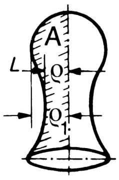
1.4.3 Volumes and surface areas of solid bodies

Table 1-25

Shape of body		V = volume	O = Surface A = Area
Solid rectangle	 	$V = a\,b\,c$	$O = 2\,(a\,b + a\,c + b\,c)$
Cube	 	$V = a^3 = \dfrac{d^3}{2.828}$	$O = 6\,a^2 = 3\,d^2$
Prism	 	$V = A\,h$	$O = U\,h + 2\,A$ A = base surface
Pyramid	 	$V = \dfrac{1}{3} A\,h$	O = A + Nappe
Cone	 	$V = \dfrac{1}{3} A\,h$	$O = \pi\,r\,s + \pi\,r^2$ $s = \sqrt{h^2 + r^2}$
Truncated cone	 	$V = (R^2 + r^2 + R\,r) \cdot \dfrac{\pi\,h}{3}$	$O = (R + r)\,\pi\,s + \pi\,(R^2 + r^2)$ $s = \sqrt{h^2 + (R - r)^2}$
Truncated pyramid	 	$V = \dfrac{1}{3} h\,(A + A_1 + \sqrt{AA_1})$	$O = A + A_1$ + Nappe
Sphere	 	$V = \dfrac{4}{3}\pi\,r^3$	$O = 4\,\pi\,r^2$
Hemisphere	 	$V = \dfrac{2}{3}\pi\,r^3$	$O = 3\,\pi\,r^2$
Spherical segment	 	$V = \pi\,h^2 \left(r - \dfrac{1}{3} h\right)$	$O = 2\,\pi\,r\,h + \pi\,(2\,r\,h - h^2) =$ $\pi\,h\,(4\,r - h)$
Spherical sector	 	$V = \dfrac{2}{3}\pi\,r^2\,h$	$O = \dfrac{\pi\,r}{2}\,(4\,h + s)$

(*continued*)

Table 1-25 (continued)

Shape of body		V = Volume	O = Surface A = Area
Zone of sphere		$V = \frac{\pi h}{3}(3a^2 + 3b^2 + h^2)$	$O = \pi (2rh + a^2 + b^2)$
Obliquely cut cylinder		$V = \pi r^2 \frac{h + h_1}{2}$	$O = \pi r (h + h_1) + A + A_1$
Cylindrical wedge		$V = \frac{2}{3} r^2 h$	$0 = 2rh + \frac{\pi}{2} r^2 + A$
Cylinder		$V = \pi r^2 h$	$O = 2 \pi r h + 2 \pi r^2$
Hollow cylinder		$V = \pi h (R^2 - r^2)$	$O = 2\pi h (R + r) + 2\pi (R^2 - r^2)$
Barrel		$V = \frac{\pi}{15} l \cdot$ $(2D^2 + Dd + 0.75 d^2)$	$O = \frac{D + d}{2} \pi d + \frac{\pi}{2} d^2$ (approximate)
Frustum		$V = \left(\frac{A - A_1}{2} + A_1\right) h$	$O = A + A_1$ + areas of sides
Body of rotation (ring)		$V = 2 \pi \varrho A$ A = cross-section	O = circumference of cross-section x $2 \pi \varrho$
Pappus' theorem for bodies of revolution		Volume of turned surface (hatched) x path of its centre of gravity $V = A\, 2 \pi \varrho$	Length of turned line x path of its centre of gravity $O = L\, 2 \pi \varrho_1$